A simple model of effort allocation

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1 Setup

A risk-neutral agent has 1 unit of effort to allocate between two tasks, each of which may either succeed or fail. Let e be the effort allocated to task 1, so that $1-e$ is the effort allocated to task 2. There is no cost of effort.

The agent gets an intrinsic pay-off θ_i if task i succeeds. Assume that $\theta_1 > \theta_2$, so that task 1 is the agent's intrinsically preferred task. There are also bonus payments, b_i, paid by a principal in the case of task success. The principal cares equally about both tasks but is unable to differentiate bonus payment across tasks, so that $b_1 = b_2 = b$.

The probability of success in task i is a concave function of effort in task i, so that the marginal return to effort in each task is positive but diminishing. For tractability I will assume here that the probability of success in each task is given by the square root of effort in that task.

Then the expected payoff for the agent is

$$(\theta_1 + b)\sqrt{e} + (\theta_2 + b)\sqrt{1-e},$$

and her problem is to maximise this with respect to effort allocation e.

2 Optimal effort allocation

The solution is

$$e^*(b) = \frac{1}{1 + \left(\frac{\theta_2 + b}{\theta_1 + b}\right)^2}.$$

In the absence of bonus pay, the agent's allocation is biased towards her intrinsically preferred task:

$$e^*(0) = \frac{1}{1 + \left(\frac{\theta_2}{\theta_1}\right)^2} > \frac{1}{2}$$

It is clear that for non-negative b, $\frac{\partial e^*}{\partial b} < 0$ and $\lim_{b \to \infty} e^* = \frac{1}{2}$. In words, the allocation of effort to the intrinsically preferred task (task 1) is decreasing in bonus pay, and for very large bonus pay the allocation is about balanced.

3 Total output

Total output is

$$\sqrt{e} + \sqrt{1-e}.$$

It is easy to show that this is at its maximum when $e = \frac{1}{2}$. Therefore, increasing the bonus pay will increase total output by reducing the distortionary effects of the agent's intrinsic bias.