1 Radon Transform of a Gaussian

Calculate the Radon transform of $p(\xi, \phi)$ of $f(x, y) = e^{-x^2-y^2}$. (Hint: there is symmetry you can exploit to simplify this problem).

2 Radon Transform of Shifted Function

Show that if the Radon transform of $f(x, y)$ is $p(\xi, \phi)$, then the Radon transform of $f(x-x_0, y-y_0)$ is $p(\xi - x_0 \cos\phi - y_0 \sin\phi)$. Also give a graphical explanation of this result. [Hint: this is somewhat easier to prove if you use the delta function form of the Radon transform given in the book: $p(\xi, \phi) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\delta(x \cos\phi + y \sin\phi - \xi)\, dxdy$.]

3 Radon transform consistency conditions

Let $p(\xi, \phi)$ be a parallel-beam sinogram and $P_\phi(\nu)$ be its 1D Fourier transform with respect to ξ for fixed ϕ, as defined in the lecture. Show that

$$P_{\phi+\pi}(\nu) = P_\phi(-\nu)$$

4 Problem 6.12 from Prince book

Consider an object comprising two small metal pellets located at $(x, y) = (2, 0)$ and $(2, 2)$ and a pice of wire stretched straight between $(0, -2)$ and $(0, 0)$.

(a) Sketch this object. Assume N photons are fired at each lateral position ℓ in a parallel-ray configuration. For simplicity, assumes that each metal object stops 1/2 the photons that are incident upon it no matter what angle it is hit.

(b) Sketch the number of photons you would expect to see as a function of ℓ for $\theta = 0°$ and $\theta = 90°$.

(c) Draw the projections you would see at $\theta = 0°$ and $\theta = 90°$.

(d) Sketh the backprojection image you would get at $\theta = 0°$ (without filtering).