First Principle of Finite Induction

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1 Introduction

In this paper, we will be doing a step by step solution to a common induction problem.Also it is show and given in many books in the study of number theory and reasoning and proofs.

2 Formula

We are given the formula

$$\sum_{n=1}^{\infty} n^2 = \frac{n(2n+1)(n+1)}{6}$$

with n element of the natural numbers.

3 Proof:

Now we begin by running off a few terms to help see the pattern emerge.

$$\sum_{n=1}^{\infty} 1^2 + 2^2 + 3^3 + 4^2 + 5^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

Let's assume that n=1

$$1 = \frac{1(2(1)+1)(1+1)}{6} = 1$$

$$1 = \frac{(2+1)(1+1)}{6} = 1$$

$$1 = \frac{6}{6} = 1$$

Next we assume that n=k and that k is an element of of the natural numbers.An we will use this equation in the Induction hypothesis that we denote as equation 1.

$$\sum_{k=1}^{\infty} 1^2 + 2^2 + 3^3 + 4^+ 5^2 + \cdots + k^2 = \frac{k(2k+1)(k+1)}{6}$$

To obtain the next term, we add k square with k+1 square to both sides of the equation.

$$\sum_{k=1}^{\infty} 1^2+2^2+3^2+4^2+5^2+\cdots+k^2+(k+1)^2 = \frac{(k+1)(2(k+1)+1)((k+1)+1)}{6}$$

Now we simplify the formula to aide in seeing the connection.

$$\sum_{k=1}^{\infty} 1^2 + 2^+3^2 + 4^2 + 5^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(2k+3)(k+2)}{6}$$

From the induction hypotheses, we use equation 1 on the left hand side (LHS) and substitute it in for the induction process.

$$\frac{k(2k+1)(k+1)}{6} + (k+1)^2 = \frac{(k+1)(2k+3)(k+2)}{6}$$

Now we factor out a (k+1) on the (LHS) of the equation.

$$(k+1)(\frac{k(2k+1)}{6}] + (k+1)) = \frac{(k+1)(2k+3)(k+2)}{6}$$

Next we combine the left hand side (LHS) equation by finding the greatest conman factor (GCF).

$$(k+1)(\frac{k(2k+1)}{6} + \frac{6(k+1)}{6}) = \frac{(k+1)(2k+3)(k+2)}{6}$$

$$(k+1)(\frac{k(2k+1)+6(k+1)}{6}) = \frac{(k+1)(2k+3)(k+2)}{6}$$

The next step is that we factor the (LHS) of the equation.

$$(k+1)(\frac{2k^2+k+6k+6}{6}) = \frac{(k+1)(2k+3)(k+2)}{6}$$

Then we combine like terms on the (LHS).

$$(k+1)(\frac{2k^2+7k+6}{6}) = \frac{(k+1)(2k+3)(k+2)}{6}$$

Now we factor the (LHS) numerator.

$$(k+1)(\frac{(2k+3)(k+2)}{6}) = \frac{(k+1)(2k+3)(k+2)}{6}$$

Final we factor the (k+1) back in the (LHS) of the equation.

$$\frac{(k+1)(2k+3)(k+1)}{6} = \frac{(k+1)(2k+3)(k+2)}{6}$$

Thus we achieved what we desired.

Reference: David M.Burton, Elements of Number Theory,page 3