In this project we want to investigate the origins of and relationships between various differentiation rules. First, all differentiation formulas and rules come from the definition. So, write down the limit definition of the derivative of f:

$$f'(x) =$$

Two early differentiation rules we learned were the Constant Rule and Constant Multiple Rule. Use the limit definition to prove that $\dfrac{d}{dx}(c) = 0$ and $\dfrac{d}{dx}\big(c \cdot f(x)\big) = c \cdot f'(x)$ for any real number constant c.

One of the next rules was the Power Rule, but only for positive integer powers. Use the limit definition to show the rule holds for the first four values of n. That is, prove that

$$\frac{d}{dx}\big(x\big) = 1 \qquad \frac{d}{dx}\big(x^2\big) = 2x \qquad \frac{d}{dx}\big(x^3\big) = 3x^2 \qquad \frac{d}{dx}\big(x^4\big) = 4x^3$$

Look in a math textbook or online and find a formula for $(a+b)^n$ where n is a positive integer. Does this formula fit the your work for $(x+h)^4$ you used above? Use this formula and the limit definition of the derivative to show the Power Rule for positive integer values of n:

$$\frac{d}{dx}\big(x^n\big) = nx^{n-1}.$$

We want to extend the Power Rule to be valid also for negative values of n. For this we need the Quotient Rule. Before we can apply the Quotient Rule, we need be absolutely sure that it is true. ☺ You see a proof of the Product Rule in the textbook, and the proof for the Quotient Rule is similar. There is a little extra trick that you can get from adding and subtracting $f(x)g(x)$ from the numerator in one step. Try the proof on your own first. Start with the definition working as much algebra as you can; you can even fill in the end of all the limit computations since you know where you want to get to. If you can connect the beginning to the end on your own, that's fabulous! If you need a little help filling in gaps, google can most definitely be your friend. Just make sure **you** understand everything you add to **your** proof.

Use the limit definition of the derivative to prove the Quotient Rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{\big(g(x)\big)^2}$$

Now, apply the Quotient Rule to show that the Power Rule holds for negative integer powers.

Finally, we want to extend the Power Rule to apply to all rational number powers. For this, we need the Chain Rule and the technique of Implicit Differentiation. You do not need to prove the Chain Rule ☺ However, it is not a bad idea to look up the proof and make sure you can understand it. Use Implicit Differentiation to show that Power Rule applies to rational powers of x.

$$\frac{d}{dx}\left(x^{m/n}\right) = \frac{m}{n}x^{m/n-1}$$

Congrats! You're a real mathematician now!