Homework 2

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November 21, 2018

1 Axioms, Interpretations, and Properties of Probability

Given an experiment and a sample space $\mathcal{S}$, the objective of probability is to assign to each event A a number $P(A)$, called the probability of event A, which will give a precise measure of the chance that A will occur.

1.1 Problem 13 (page 62)

A computer consulting firm presently has bids out on three projects. Let A_i= {awarded project i}, for $i = 1, 2, 3$, and suppose that $P(A_1) = .22$, $P(A_2) = 0.25$, $P(A_3) = .28$, $P(A_1 \cap A_2) = .11$, $P(A_1 \cap A_3) = 0.05$, $P(A_2 \cap A_3) = 0.07$, $P(A_1 \cap A_2 \cap A_3) = 0.01$. Express in words each of the following events, and compute the probability of each event:

1.2 13a

$A_1 \cup A_2$

1.3 Solution to 13a

1.4 13b

$A_1^c \cap A_2^c$

$$Hint : (A_1 \cup A_2)^c = A_1^c \cap A_2^c$$

1.5 Solution to 13b

1.6 13c

$A_1 \cup A_2 \cup A_3$

1.7 Solution to 13c

1.8 13d

$A_1^c \cap A_2^c \cap A_3^c$

1.9 Solution to 13d

1.10 13e

$A_1^c \cap A_2^c \cap A_3$

1.11 Solution to 13e

1.12 13f

$(A_1^c \cap A_2^c) \cup A_3$

1.13 Solution to 13f

2 Counting Techniques

When the various outcomes of an experiment are equally (the same probability is assigned to each simple event), the task of computing probabilities reduces to counting. Letting N denote the number of outcomes in a sample space and $N(A)$ represent the number of outcomes contained in an event A,

$$P(A) = \frac{N(A)}{N}$$

If a list of the outcomes is easily obtained and N is small, then N and $N(A)$ can be determined without the benefit of any general counting principles.

2.1 Problem 38 (page 72)

A box in a certain supply room contains four 40-W light bulbs, five 60-W bulbs, and six 75-W bulbs. Suppose that three bulbs are randomly selected.

2.2 38a

What is the probability that exactly two of the selected bulbs are rated 75-W?

2.3 Solution to 38a

2.4 38b

What is the probability that all three of the selected bulbs have the same rating?

2.5 Solution to 38b

2.6 38c

What is the probability that one bulb of each type is selected?

2.7 Solution to 38c

2.8 38d

Suppose now that bulbs are to be selected one by one until a 75-W bulb is found. What is the probability that it is necessary to examine at least six bulbs?

2.9 Solution to 38d

3 Conditional Probability

The probabilities assigned to various events depend on what is known about the experiemental situation when the assignment is made. Subsequent to the initial assignment, partial information relevant to the outcome of the experiment may become available. We will use the notation $P(A|B)$ to represent the conditional probability of A given that the event B has occurred.

3.1 Problem 58 (page 81)

Show that for any three events A,B, and C with $P(C) > 0$, $P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$.

3.2 Solution to 58

4 Independence

The definition of conditional probability enables us to revise the probability $P(A)$ originally assigned to A when we are subsequently informed that another event B has occurred; the new probability of A is $P(A|B)$. Two events A and B are independent if $P(A|B) = P(A)$ and are dependent otherwise.

4.1 Problem 73 (page 86)

If A and B are independent events, show that A^c and B are also independent. Hint: First establish a relationship between $P(A^c \cap B$, $P(B)$, and $P(A \cap B)$.

4.2 Solution to 73

5 Discrete Random Variables and Probability Distributions

The probability distribution or probability mass function (pmf) of a discrete rv is defined for every number x by $p(x) = P(X = x) = P(alls \in S : X(s) = x)$. Suppose $p(x)$ depends on a quantity that be assigned any one of a number of possible values, with each different value determining a different probability distribution.

5.1 Problem 15 (page 105)

Many manufacturers have quality control programs that include inspection of incoming materials for defects. Suppose a computer manufacturers receives computer boards in lots of five. Two boards are selected from each lot for inspection. We can represent possible outcomes of the selection process by pairs. For example, the pair (1,2) represents the selection of boards 1 and 2 for inspection.

5.2 15a

List the 10 different possible outcomes.

5.3 Solution to 15a

5.4 15b

Suppose that boards 1 and 2 are the only defective boards in a lot of five. Two boards are chosen at random. Define X to be the number of defective boards observed among those inspected. Find the probability distribution of X.

5.5 Solution to 15b

5.6 15c

Let $F(x)$ denote the cdf of X. First determine $F(0) = P(X \leq 0)$, $F(1)$, and $F(2)$; then obtain $F(x)$ for all other X.

5.7 Solution to 15c

5.8 Problem 20 (page 105)

Three couples and two single individuals have been invited to an investment seminar and have agreed to attend. Suppose the probability that any particular couple or individual arrives late is 0.4 (a couple will travel together in the same vehicle, so either both people will be on time or else both will arrive late). Assume that different couples and individual are on time or late independently of one another. Let X= the number of people who arrive late for the seminar,

5.9 20a

Determine the probability mass function of X. Hint: label the three couples 1, 2, and 3 and the two individuals 4 and 5.

5.10 Solution to 20a

5.11 20b

Obtain the cumulative distribution function of X. and use it to calculate $P(2 \leq X \leq 6)$.

5.12 Solution to 20b