Proposition R.231: Prove that $A = \left\{m + n\sqrt{3} \mid m, n \in \mathbb{Z}\right\}$ is closed under multiplication.

Proof: Let $A = \left\{m + n\sqrt{3} \mid m, n \in \mathbb{Z}\right\}$, and let $m + n\sqrt{3}$ and $p + q\sqrt{3}$ be elements of A. See Figure 1 to see what a table looks like. Then

$$\left(m + n\sqrt{3}\right)\left(p + q\sqrt{3}\right) = mp + mq\sqrt{3} + np\sqrt{3} + 3qn \tag{1}$$
$$= (mp + 3qn) + (mq + np)\sqrt{3}. \tag{2}$$

Since $m, n, p, q \in \mathbb{Z}$, $mp + 3nq$ and $mq + np$ are both integers. Therefore,

$$\left(m + n\sqrt{3}\right)\left(p + q\sqrt{3}\right) \in A,$$

and A is closed under multiplication. $\square$

A	B	If A then B.
True	True	
True	False	
False	True	
False	False	

Figure 1: And here is a table inserted for no reason whatsover