

# Homework X

Your Name

February 22, 2016

## Problem 33.13

As a comet swings around the Sun, ice on the comet's surface vaporizes, releasing trapped dust particles and ions. The ions, because they are electrically charged, are forced by the electrically charged solar wind into a straight ion tail that points radially away from the Sun. The (electrically neutral) dust particles are pushed radially outward from the Sun by the radiation force on them from sunlight. Assume, that the dust particles are spherical, have density  $3.5 \times 10^3 \text{ kg/m}^3$ , and are totally absorbing. (a) What radius must a particle have in order to follow a straight path, like path 2 in the figure? (b) If its radius is larger, does it path curve away from the Sun (like path 1) or toward the Sun (like path 3)?

## Problem 33.71 — Rainbows

A light ray enters and then leaves a falling, spherical raindrop after one internal reflection. The final direction of travel is deviated (turned) from the initial direction of travel by angular deviation  $\theta_{\text{dev}}$ .

1. Show that  $\theta_{\text{dev}} = 180^\circ + 2\theta_i - 4\theta_r$
2. Using Snell's law, substitute for  $\theta_r$  in terms of  $\theta_i$  and the index of refraction of the water. Then graph  $\theta_{\text{dev}}$  vs.  $\theta_i$  for the range of possible  $\theta_i$  values and for  $n = 1.331$  for red light and  $n = 1.333$  for blue light.
3. Determine the angle of minimum deviation for the red light
4. and the blue light.

5. If these colors from the inner and outer edges of a rainbow, what is the angular width of the rainbow?
- (a) Show that  $\theta_{\text{dev}} = 180^\circ + 2\theta_i - 4\theta_r$ .
- (b) Using Snell's law, substitute for  $\theta_r$  in terms of  $\theta_i$  and the index of refraction of the water. Then graph  $\theta_{\text{dev}}$  vs.  $\theta_i$  for the range of possible  $\theta_i$  values and for  $n = 1.331$  for red light and  $n = 1.333$  for blue light.
- (c) Determine the angle of minimum deviation for the red light
- (d) and the blue light.
- (e) If these colors from the inner and outer edges of a rainbow, what is the angular width of the rainbow?

### Problem 33.63

We have a simple model of an optical fiber: a plastic core ( $n_1 = 1.58$ ) surrounded by a plastic sheath ( $n_2 = 1.53$ ). A light ray enters the fiber at angle  $\theta$ . What is the maximum value of  $\theta$  that allows total internal reflection at point A?

**Solution** The light ray enters the fiber at angle  $\theta$ , it is refracted to angle  $\theta_r$  and travels until it reaches point A (the point where it reaches the side of the core and the boundary with the sheath). At point A we want all of the light to be reflected back into the core.

Since the line going through the axes is parallel to the sides of the cylinder/optical fiber,  $\theta_r$  is the same as the angle between the side of fiber and the light ray at point A. Thus the incident angle (between the light ray and the normal) is  $90^\circ - \theta_r$ . Since none of the light will pass into the sheath, that angle is  $90^\circ$ . We use Snell's law to find the maximum  $\theta_r$  that results in total internal reflection. ( $90^\circ - \theta_r$  is the minimum angle from the normal to experience total internal reflection, thus the complement  $\theta_r$  is at a

maximum.)

$$\begin{aligned}n_1 \sin(90^\circ - \theta_r) &= n_2 \sin 90^\circ \\n_1 \cos \theta_r &= n_2 \cdot 1 \\ \cos \theta_r &= \frac{n_2}{n_1} = \frac{1.53}{1.58} = 0.968 \\ \theta_r &= 14.5^\circ\end{aligned}$$

Let's use Snell's law again at the boundary between the air and the core to find the maximum incident angle  $\theta_i$

$$\begin{aligned}n_{\text{air}} \sin \theta_i &= n_1 \sin \theta_r \\ 1 \cdot \sin \theta_i &= 1.58 \sin 14.5^\circ \\ \theta_i &= \boxed{23.6^\circ}\end{aligned}$$

To continue having total internal reflection, the maximum angle the light ray can enter with is  $23.6^\circ$ .